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# A deformable model for fingerprint matching

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#### Abstract

The process of automatic fingerprint matching is affected by the nonlinear deformation introduced in the image during fingerprint sensing. Given several template impressions of a finger, we estimate the "average" deformation of each template impression by comparing it with the rest of the impressions of that finger. The average deformation is developed using the thin plate spline (TPS) model and is based on minutia point correspondences between pairs of fingerprint impressions. The estimated average deformation is utilized to pre-distort the minutiae points in the template image before matching it with the minutiae points in the query image. We show that the use of an average deformation model leads to a better alignment between the template and query minutiae points. An index of deformation is proposed for choosing the deformation model with the least variability arising from a set of template impressions corresponding to a finger. Our experimental data consists of 1600 fingerprints corresponding to 50 different fingers collected over a period of 2 weeks. It is shown that the average deformation model leads to an improvement in the alignment between impressions originating from the same finger. © 2004 Pattern Recognition Society. Published by Elsevier Ltd. All rights reserved.

Keywords: Fingerprints; Non-linear deformation; Thin plate splines; Template selection; Index of deformation

# 1. Introduction

The uniqueness of a fingerprint is determined by the topographic relief of its ridge structure and the presence of ridge anomalies, termed as minutiae points. The problem of automatic fingerprint matching involves determining a measure of similarity between two fingerprint impressions by comparing their ridge structure and/or the spatial distribution of the minutiae points [1–4]. The image acquisition process, however, introduces non-linear distortions in the ridge structure and, consequently, in the spatial location of minutiae

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points, thereby confounding the matching process. This distortion is a function of several parameters including the orientation of the sensor with respect to the finger, the amount of pressure applied by the subject, the disposition of the subject (sitting or standing), the motion of the finger prior to its placement on the sensor, the moisture content of the skin (dry, oily or wet), the elasticity of the skin, etc. Therefore, the distortions observed in a fingerprint vary from one acquisition to the next. For reliable matching, these nonlinear distortions must be accounted for, prior to comparing two fingerprint images. Deformation models based on affine transformations invariably lead to unsatisfactory matching results since the distortions are basically elastic in nature (Fig. 1).

In this paper we develop an average deformation model for a fingerprint impression by comparing it with several

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Fig. 1. Aligning two impressions of the same finger using an affine transformation. Due to non-linear distortions, the alignment is not accurate in some regions. Only fingerprint ridges are shown for clarity.

other impressions<sup>1</sup> of the same finger. The average deformation model is then used to distort the minutiae set of a template impression *before* comparing it with the minutiae set of a query impression acquired during verification. Our thesis is that pre-distorting the template minutiae set aids in a better alignment of the query and template minutiae sets. To restrict the number of parameters affecting fingerprint distortion, the fingerprint data used in our experiments were obtained from subjects who adopted a sitting posture while interacting with a sensor that was horizontally situated. The subjects were not aware of the purpose of the data acquisition and, therefore, no deliberate attempts were made to introduce distortions during image acquisition.

There have been several attempts to account for the elastic distortions in fingerprint images. Cappelli et al. [5] used a combination of rigid and non-rigid transformation techniques to model non-uniform distortions in three different concentric regions around the center of the fingerprint image. However, their model was not used for fingerprint matching. Kovács-Vajna [6] used a triangular matching algorithm to compare two sets of minutiae points and to account for large-scale deformations. Watson et al. [7] constructed distortion tolerant filters for each (template) fingerprint prior to performing a correlation type matching. There have also been efforts to model the non-linear distortion effects locally (for example, Refs. [8,9]).

Fig. 2. Three impressions of the same finger exhibiting different non-linear distortions.

Warping methods can be used to obtain global deformation models for purposes of image registration. Applications of warping techniques abound in the statistical, medical imaging and computer vision literature. There have been a variety of image registration techniques motivated from different principles; examples include warping by elastic deformations [10,11], optical or fluid flow [12,13], diffusion processes [14], Bayesian prior distributions [15], and thin-plate splines (TPS) [16-18]. Only recently have warping techniques based on deformation models been used to model distortions in fingerprint images for the purpose of matching [19,20]. Warping enables the distortions to be estimated and subsequently removed prior to matching. It is shown in [20] that this procedure results in superior matching performance compared to algorithms which either do not model distortions or model them using rigid transformations. In both Refs. [19,20], thin-plate splines is used as the distortion model.

When multiple impressions (templates) of a finger are available, it is observed that the non-linear distortion present in them vary significantly (Fig. 2). Further, the *relative* distortion between two pairs of fingerprint impressions, that share a common impression, is different (Fig. 3). In this paper, we develop an *average* deformation model for the templates of a single finger together with an estimate of its variability based on TPS. An index of deformation is suggested as a means of selecting that template fingerprint impression with the least variability. The average deformation model can be incorporated into the matching algorithm when comparing the template impression with a query image.

<sup>&</sup>lt;sup>1</sup> We assume, therefore, that multiple impressions of a user's fingerprint are made available either at the time of enrolment or over a period of time.



Fig. 3. Non-linear deformations (with rotation and translation parameters removed) associated with two pairings involving the same template: (a) template image; (b) and (c) query images; (d) and (e) non-linear deformation of (a) into (b) and (c), respectively.

Experimental results indicate that a better alignment is achieved by incorporating finger-specific deformation models in the verification mode.

The method presented in this paper significantly differs from the technique suggested by Bazen et al. [20]. The authors, in their approach, use a TPS model to align *every pair* of impressions, even if they are from two different fingers. This forces an alignment between impressions originating from two different fingers resulting in a higher false accept rate (FAR). The average deformation model that we propose, on the other hand, is computed using fingerprint impressions originating from the same finger. In the verification stage, if the observed deformation (between the query and template minutiae points) is not consistent with this typical deformation, then the matching score will be low.

The rest of the paper is organized as follows: Section 2 describes the thin plate spline model that we have adopted and its applicability to the average deformation model for fingerprints; Section 3 shows how the average deformation model can be incorporated into a fingerprint matching sys-

tem; Section 4 introduces the index of deformation; Section 5 presents the experimental results; finally, Section 6 provides a summary of the paper and directions for future work.

# 2. General methodology

Let  $I_0$  and  $I_1$  denote two fingerprint impressions corresponding to the same finger. A warping of  $I_0$  to  $I_1$  is defined as the function F such that

$$F(I_0) = I_1.$$
 (1)

The function *F* is called the warping function which takes  $I_0$  to  $I_1$ . The present application registers the two impressions  $I_0$  and  $I_1$  by matching the locations of their minutiae points. Consequently, we will only consider warping techniques that register impressions in terms of two sets of point patterns (the two sets of minutiae points in this case). Thus, in Eq. (1), the warping function can be thought of as  $F: S \to S, S \subset \mathbb{R}^2$ , that registers two sets of minutiae points derived from  $I_0$  and  $I_1$ . More specifically, consider a pair of point patterns with known correspondences, say,  $U = (u_1, u_2, ..., u_m)^T$  and  $V = (v_1, v_2, ..., v_m)^T$  that need to be registered; here,  $u_k$  and  $v_k$  denote the locations (represented as vectors in  $R^2$ ) of the kth corresponding pair and m is the total number of corresponding points. We assume that the two point pattern sets with known correspondences are aligned as close as possible using a rigid transformation model prior to non-linear warping. This can be achieved using the Procrustes analysis (see Ref. [21], for example). Thus, a warping function, F, that warps U to V subject to perfect alignment is given by the conditions

$$F(u_j) = v_j$$
 for  $j = 1, 2, ..., m$ . (2)

Suppose now we have *N* such pairs of correspondences given by  $(U_i, V_i)_{i=1}^N$ , where  $U_i = (u_{i,1}, u_{i,2}, \dots, u_{i,m_i})^T$ ,  $V_i = (v_{i,1}, v_{i,2}, \dots, v_{i,m_i})^T$  and  $m_i$  is the number of point patterns in the *i*th pair. In this case, the *i*th warping function,  $F_i$ , with  $F_i : S \to S$  satisfies

$$F_i(u_{i,j}) = v_{i,j} \tag{3}$$

for  $j = 1, 2, ..., m_i$  and i = 1, 2, ..., N. The average deformation at each point  $u \in S$  is defined as

$$\bar{F}(u) = \frac{1}{N} \sum_{i=1}^{N} F_i(u)$$
(4)

and the covariance matrix corresponding to the deformation is given by

$$D_F(u) = \frac{1}{N} \sum_{i=1}^{N} (F_i(u) - \bar{F}(u)) \cdot (F_i(u) - \bar{F}(u))^{\mathrm{T}}.$$
 (5)

We use TPS to estimate the deformation  $F_i$  for the *i*th pair  $(U_i, V_i)$ . TPS represents a natural parametric generalization

from rigid to mild non-rigid deformations. The deformation model is given in terms of the warping function  $F_i(u)$ , with

$$F_i(u) = c_i + A_i \cdot u + W_i^{\mathrm{T}} s(u), \tag{6}$$

where  $u \in S$ ,  $c_i$  is a 2 × 1 vector,  $A_i$  is a 2 × 2 matrix,  $W_i^T$  is a  $m_i \times 2$  matrix,  $s(u) = (\sigma(u - u_{i,1}), \sigma(u - u_{i,2}), \dots, \sigma(u - u_{i,m_i}))^T$  and

$$\sigma(u) = \begin{cases} ||u||^2 \log(||u||) & ||u|| > 0, \\ 0 & ||u|| = 0. \end{cases}$$
(7)

In Eq. (6), there are 6 and  $2m_i$  parameters corresponding to the rigid and non-rigid parts of the deformation model, respectively, resulting in a total of  $2m_i + 6$  parameters to be estimated. However, Eq. (3) only provides  $2m_i$  constraints. For the coefficients to be uniquely estimated, we further assume that the coefficients  $W_i$  satisfy (i)  $1_{m_i}^T W_i = 0$  and (ii)  $U_i^T W_i = 0$ , where  $1_{m_i}$  is the vector of ones of length  $m_i$ . Thus, the coefficients of the *i*th TPS model can be obtained from the matrix equation

$$\begin{bmatrix} H & 1_{m_i} & U_i \\ 1_{m_i}^{\mathrm{T}} & 0 & 0 \\ U_i^{\mathrm{T}} & 0 & 0 \end{bmatrix} \begin{bmatrix} W_i \\ c_i^{\mathrm{T}} \\ A_i^{\mathrm{T}} \end{bmatrix} = \begin{bmatrix} V_i \\ 0 \\ 0 \end{bmatrix},$$
(8)

where  $(H)_{il}$  is the  $m_i \times m_i$  matrix with entries  $\sigma(u_{i,i} - u_{i,l})$ .

The matrix equation in (8) gives rise to a TPS model that minimizes the bending energy subject to the perfect alignment constraints in Eq. (3). A more robust TPS model can be obtained by relaxing the constraints in Eq. (3), and instead determining an  $F_i$  which minimizes the expression

$$\sum_{j=1}^{m_i} (v_{i,j} - F_i(u_{i,j}))^{\mathrm{T}} (v_{i,j} - F_i(u_{i,j})) + \lambda J(F_i), \qquad (9)$$

where

$$J(F_i) = \sum_{j=1}^{2} \int_{S} \left\{ \left( \frac{\partial^2 F_{i,j}(x, y)}{\partial x^2} \right)^2 + 2 \left( \frac{\partial^2 F_{i,j}(x, y)}{\partial x \partial y} \right)^2 + \left( \frac{\partial^2 F_{i,j}(x, y)}{\partial y^2} \right)^2 \right\} dx dy$$
(10)

represents the bending energy associated with  $F_i = (F_{i,1}, F_{i,2})^{\mathrm{T}}$ ,  $F_{i,j}$  is the *j*th component of  $F_i$ , and  $\lambda > 0$ . The case  $\lambda = 0$  gives rise to the TPS model described by Eq. (8). For general  $\lambda > 0$ , the coefficients of the resulting TPS model can be obtained using Eq. (8) with *H* replaced by  $H + \lambda I_{m_i}$ , where  $I_{m_i}$  is the  $m_i \times m_i$  Identity matrix.

The average deformation defined in Eq. (4) can be interpreted as the typical deformation that arises when a specific finger is placed on the sensor. For a different finger, we observe an average deformation that is different. Fig. 4 shows the average deformation for three different fingers; it can be clearly seen that the average warping functions are different for the three fingers and hence we conclude that the fingerprint deformation is finger-specific.



Fig. 4. The average deformation model of three different fingers shown as distortions on a reference grid. (a), (c) and (e) are the original impressions while (b), (d) and (f) are the corresponding average deformation models.

The covariance matrix corresponding to the deformations in Eq. (5), defined at each pixel *s*, is a measure of the variability associated with the estimated warping functions around the average. We consider pixel-wise measures of the extent of variability present in the covariance matrix in terms of a matrix function  $\phi$ . Two choices for  $\phi$  are (i) the determinant,  $\phi(D) = |D|$ , and (ii) the trace,  $\phi(D) = \text{tr}(D)$ . Pixels with large (small) values of  $\phi$  indicate high (low) variability in the deformation that warps one set of point patterns to another. It was observed that there is a larger variability of the deformation at the boundaries of *S* compared to the center.

### 3. Average deformation for authentication

We now illustrate how to incorporate the average deformation model in the authentication process when a query

fingerprint image O, is presented to the system along with a claimed identity. The authentication proceeds by comparing O with the template T, corresponding to the claimed identity. This is done in the following three steps: (i) the minutiae points in both the query and template fingerprint images are extracted; (ii) the locations of the minutiae points in the template are warped into another set of points using the average deformation model specific to the claimed identity; (iii) the warped points are matched with the set of minutiae points in the query image, and the number of matches is recorded. Specifically, if  $M_T$  and  $M_O$  denote the locations of the template and query minutiae points, respectively, and  $\bar{F}_T$  denotes the average deformation model corresponding to  $M_T$ , then the warped template points are given by  $\overline{F}(M_T)$  and a matching score for comparing  $\overline{F}(M_T)$  and  $M_O$  is given by  $p = n^2/n_T n_Q$ , where *n* is the number of matches between  $\overline{F}(M_T)$  and  $M_O$ , and  $n_T$   $(n_O)$  is the number of minutiae points in  $M_T$  ( $M_O$ ). Large values of p (close to 1) will lead to positive authentication whereas small values of p (close to 0) will lead to a rejection by the system.

In order to determine the number of matching pairs between two minutiae sets, we adopt the following rather simple technique. We first select a reference minutiae pair (one minutia from each set), and use them to compute the translational offsets necessary to align the two sets. The translated query minutiae set is then rotated about it's reference minutiae such that a large number of its minutiae points are paired with those in the template set within a 5-pixel (radius) tolerance window. We repeat this by considering all possible pairings of minutiae in the two sets, and reporting that pair which results in the maximum number of matched minutiae points. More sophisticated matchers exist in the literature, but to demonstrate the effectiveness of the proposed model, the above matching technique suffices. The goal of our technique is to pre-distort the template minutiae set, thereby improving the *alignment* between the template and the query minutiae sets.

#### 4. Index of deformation

Suppose we have *N* impressions,  $T_1, T_2, \ldots, T_N$ , of a finger. Which one of these images should we consider as the reference image for estimating the average deformation for this finger? To address this, we consider all possible pairings of the type  $(T_i, T_j)$  where  $i \neq j$ . Thus, for fixed *i*, there are a total of N - 1 pairings giving rise to N - 1 pairs of correspondences:  $(T_i, T_j), j \neq i$ . With each  $T_i$  as the reference, we warp  $T_i$  to each  $T_j, j \neq i$  using the methodology outlined in Section 2. The average deformation  $\overline{F}_{T_i}$  and the covariance matrix  $D_{\overline{F}_{T_i}}$  (Eqs. (4) and (5), respectively) are then obtained. We define the *i*th index of deformation  $\Phi_i$ , as

$$\Phi_i = \frac{1}{|S|} \sum_{u=1}^{|S|} \phi(D_{\bar{F}_{T_i}(u)}), \tag{11}$$

where,  $\phi(D) = \operatorname{tr}(D)$ , and |S| is the number of pixels in the domain *S*. Subsequently, we choose  $T_k$  as the template with the smallest variability in deformation if  $k = \arg \min_i \Phi_i$ . In effect, we choose that template  $T_k$  that minimizes the average variation across pixels measured in terms of  $\Phi_i$ . Low (high) values of the index of deformation indicate that the warping functions are similar (dissimilar) to each other. It is imperative that "outlier" deformations are not present in this representative set.

# 5. Experimental results

In order to apply the TPS model to reliably estimate fingerprint deformation, we need to have several impressions of the same finger (atleast 10). Large number of impressions of a finger are not available in standard fingerprint databases (e.g., FVC 2002 [22]). Therefore, fingerprint images of 50 fingers (five subjects) were acquired using the Identix sensor  $(256 \times 255, 380 \, \text{dpi})$  over a period of 2 weeks. The subjects did not deliberately distort their fingerprints during the time of image acquisition. There were 32 impressions corresponding to every finger, resulting in a total of 1600 impressions. One half of the impressions (16 for each finger, resulting in 800 impressions) were used as templates to compute the average deformation model for each finger, while the remaining 800 impressions were used for testing. For each template image, T, the minutiae set,  $M_T$ , was extracted using the algorithm outlined in [4]. The average deformation model of T, was obtained using the methodology described in Section 2 based on pairings with the remaining 15 impressions of the same finger (Eq. (9) with  $\lambda = 5$ ). The correspondences were discovered using the linear pairing algorithm given in Ref. [4]. The minutiae set  $M_T$  was transformed to the deformed set,  $MD_T \equiv \overline{F}_T(M_T)$  using  $F_T$ . A total of 800 sets (50  $\times$  16) of deformed minutiae points were thus obtained. In order to test the matching performance of the deformed minutiae sets, and the utility of the index of deformation,  $\Phi$ , the following two experiments were conducted.

In the first experiment, the matching performance of the deformed minutiae template sets was evaluated. Every template image, T, was compared with every query image, Q, and two types of matching scores were generated for each comparison: the matching score obtained by matching (i)  $M_T$  with  $M_Q$ , and (ii)  $MD_T$  with  $M_Q$ . The Receiver Operating Characteristic (ROC) curve plotting the genuine accept rate (GAR) against the false accept rate (FAR) at various matching thresholds is presented in Fig. 5. An overall improvement is observed when the average deformation model is used to distort  $M_T$  prior to matching. Fig. 6 shows the improved registration between the query and the template minutiae sets when the average deformation model is applied prior to the rigid transformation.

In the second experiment, the advantage of using the index of deformation is demonstrated. The  $\Phi$ -index of



Fig. 5. Improvement in matching (alignment) performance using the average deformation model: distorted minutiae set ( $(\Box)$ ) and non-distorted minutiae set ( $(\circ)$ ). A simple minutiae matcher has been used to demonstrate the improvement in alignment.

deformation (with  $\phi(D) = \operatorname{tr}(D)$ ) of every template image is used to rank the templates according to their variability in the distortion. Fig. 7 shows the average deformation model corresponding to the 16 templates of a finger sorted in increasing  $\Phi$ -values. The template images can now be split into two sets: (i) impressions with the least  $\Phi$  values for every finger (the  $\Phi$ -optimal templates) and (ii) the remaining impressions for every finger (the  $\Phi$ -suboptimal templates). We repeated the matching procedure outlined above using these two template sets. The resulting ROC curve is shown in Fig. 8. From the figure, it is clear that using  $\Phi$ -optimal templates results in better performance compared to using  $\Phi$ -suboptimal templates. Further, the  $\Phi$ -suboptimal templates still yield better performance compared to the non-distorted templates, thus demonstrating the importance of the average deformable model.

### 6. Summary and future work

We have presented an average deformation model for fingerprints based on thin plate splines (TPS). The average deformation model corresponding to a template fingerprint is shown to improve the alignment of the minutiae points between the template and query minutiae sets. We have also proposed an index of deformation to aid in the selection of the template with the smallest variation in deformation from a set of impressions obtained from the same finger. Experimental results indicate that incorporating the finger-specific deformation model in the matching stage improves the alignment between minutiae sets. The technique presented here uses correspondence between minutiae points of two images to compute the average deformation model. These correspondences are automatically



Fig. 6. Improved alignment using the average deformation model of three different query and template minutiae pairs. The arrows indicate significant improvement in point correspondences. The matching score improved from 0.300 (affine model only) to 0.408 (average deformation model) in (a), from 0.114 to 0.145 in (b), and from 0.300 to 0.481 in (c). The matching score in (b) is low because of the small amount of overlap between the query and the template.



Fig. 7. The average deformation model (shown as deformations on a reference grid) corresponding to the 16 templates of a finger sorted in increasing  $\Phi$ -values, (a) is chosen to be the optimal template since it has the least  $\Phi$ -value.

detected and are, therefore, prone to error. Currently, we are working on using the ridge structure of the fingerprint to develop more robust correspondences between image pairs. We are also acquiring fingerprint impressions of 100 fingers over a period of 2 months to test the model on a larger database.



Fig. 8. Improvement in alignment using the average deformation model and the index of distortion:  $\Phi$ -optimal templates only (' $\diamond$ '),  $\Phi$ -suboptimal templates (' $\Box$ ') and non-distorted templates (' $\circ$ ').

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